# Final QCD results from LEP

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Recent results on QCD studies in $e^+e^-$ annihilations at LEP are presented. Data recorded by the LEP experiments at centre-of-mass energies between 91.2 to 206 GeV are included. The main topic is the measurement of $\alpha_s$ from event shape variables and associated aspects like the energy evolution or non-perturbative power-law corrections. These 'standard' measurements are complemented by new determinations using the 4-jet rate with an excellent precision. A summary of the results on QCD colour factors from angular correlations in the 4-jet system completes this report.

## 1. Introduction

The LEP experiments ALEPH, DELPHI, L3 and OPAL have collected 700 $\text{pb}^{-}1$ of annihilation data from 1989 to 2000. Final QCD analyses of these data using the latest detector simulation and advanced correction techniques are now being published. The experiments presented cumulative summary papers including various centre-of-mass energies and a wide collection of observable. The experimental systematic uncertainties are for many observables as small as 1%, and the statistical errors typically 0.1 %. Hence, these data of un-preceeded precision will serve as reference for future experiments. This report summarises recent measurements of $\alpha_s$ from event shapes and 4-jet observables, investigations of power law corrections and results on QCD colour factors.

## 2. Measurements of $\alpha_s$ from event shapes

Theoretical calculations for event-shape distributions are available at next-to-leading order (NLO) complemented by all-orders resummation of large leading and sub-leading logarithms (NLLA) for certain observables. A unified prescription for the matching of fixed-order and resummed calculations, the so-called modified logR matching scheme, has recently been suggested [1] and is applied by the LEP experiments for their analysis. The most commonly used variables are thrust, heavy jet mass, wide and total jet broadenings, C-parameter and $y_3$, the 3-jet resolution parameter in Durham scheme. These observables were also selected by the LEP QCD working group for a LEP combination [2]. A virtue of event-shape distributions compared to other observables is that even with limited event statistics at LEP2 energies a measurement of $\alpha_s$ is possible with a reasonable precision and enables the observation of the energy evolution of $\alpha_s$. An example of a final analysis is shown in fig. 1 where the measurements of the wide jet broadening variable at all LEP energies are compared to the result of fits with the NLO+NLLA theoretical prediction. ALEPH has in addition applied a perturbative NLO correction for the b-quark mass. In general the description of the data is good, although restricted to the central part of the distributions at LEP1 where the high precision of the data requires NNLO calculations. The measurements have been corrected for acceptance and detector resolution effects and the fits are carried out at hadron level. Therfore, the perturbative prediction is folded to the hadron level by means of transition matrix accounting for the hadronisation, obtained from standard QCD generators like PYTHIA, HERWIG or ARIADNE. The systematic uncertainties for the measurement of $\alpha_s$ is dominated by theoretical uncertainties induced by missing higher orders. The LEP QCD working group advises the recommendation of [1] for the estimation of perturbative uncertainties. This method combines different estimates in the

*Contribution to the Proceedings of 'QCD 04 International Conference in Quantum Chromodynamics', Montpellier, France, July 5 - 10, 2004

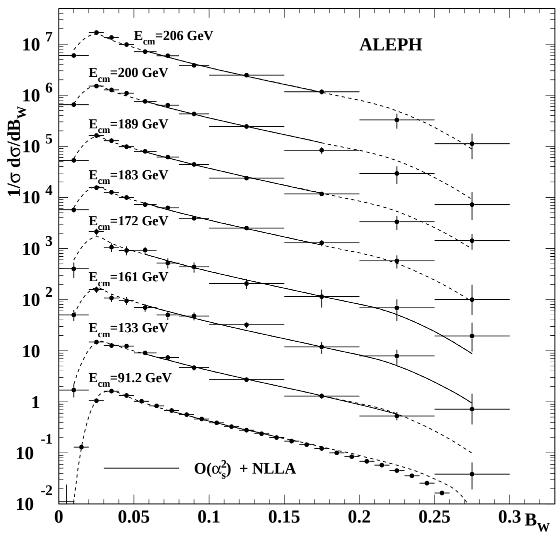


Figure 1. Distributions at various centre-of-mass energies of the wide jet broadening compared to theoretical predictions used to determine $\alpha_s$.

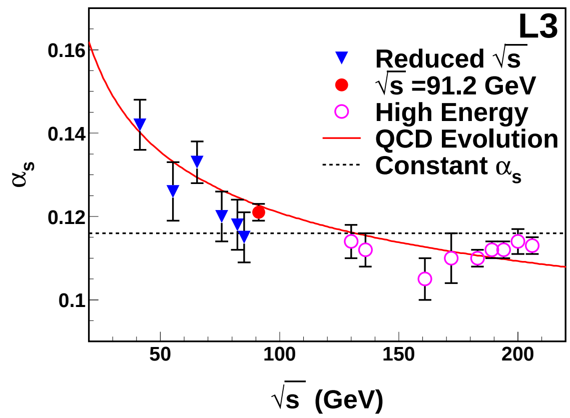


Figure 2. Measurements of $\alpha_s$, combining five event-shape variables, as function of centre-of-mass energy.

'uncertainty band method', which takes not only standard renormalisation scale variations but also a variation of $x_L$, the resummed logarithmic variable re-scaling factor, into account [3].

### 2.1. Combined measurements

The measurements using different variables and different energies are combined in single numbers per $E_{\mathrm{CM}}$ and finally using the QCD-predicted evolution in global result for $\alpha_s(M_Z)$. In fig. 2 the combined result from L3 for $\alpha_s(Q)$ is shown, including also measurements using radiative events resulting in reduced centre-of-mass energies below $M_{\mathrm{Z}}$. The experiments have applied different techniques for their combinations in terms of the correlation of systematic uncertainties, but in all cases the theoretical uncertainties appear to be largely correlated, both between different variables and between different energies. As a consequence, the gain in precision of the combined measurements in limited and the combined uncertainty appears as average rather than an improved uncertainty.

ALEPH has carried out a global analysis of 6 variables at 8 energies between 91 and 206 GeV [4], based on an integrated luminosity of about 700 pb$^{-1}$. Their combined result for $\alpha_s(M_Z)$ is a weighted average with weights proportional to the inverse square of the total individual errors. To account for correlations of systematic uncertainties the whole combination is repeated separately for all the individual variations of the analysis. The final result is

$$\alpha_s(M_{\mathrm{Z}}) = 0.1214 \pm 0.0014_{exp} \pm 0.0046_{th}.$$

L3 presented in a collective preprint [5] various studies of QCD, including measurements of $\alpha_s$ from an earlier publication [6]. They combined the same five variables, but excluded $-\ln y_3$. Measurements at reduced centre-of-mass energies using radiative events were included. Theoretical uncertainties are estimated by a variation of the renormalisation scale and different matching schemes. L3 has chosen at LEP2 very large fit ranges, in contrast to all other experiments, in order to reduce the statistical uncertainty. This leads on the other side to larger theoretical errors. The combination method proceeds in two steps: first an unweighted average of the five variables is build at each $E_{\mathrm{CM}}$, second these combined

measurements are fit by the QCD evolution for $\alpha_s(M_Z)$. At that stage correlations are included by the assumption of minimal overlap of systematic uncertainties. The result of this combination is:

$$\alpha_s(M_{\mathrm{Z}}) = 0.1227 \pm 0.0012_{exp} \pm 0.0058_{th}.$$

DELPHI updated a publication [7] on event-shapes with a new re-analysis [8] according to the LEP QCD scheme. Traditionally DELPHI presents measurements based on three perturbative methods: the standard modified logR matching scheme, the pure NLLA calculation (valid only in a narrow 2-jet region) and the fixed order prediction alone with an optimised scale $x_\mu^{opt}$. The perturbative uncertainties are estimated by a variation of $x_L$ only for the logR/NLLA schemes and by a renormalisation scale variation of a factor of two around the optimum $x_\mu^{opt}$. The same set of variables as used by L3 is combined at energies between 89 and 207 GeV, hence including also off-peak data at 89 and 93 GeV separately. The combination technique applied by DELPHI follows closely the LEP QCD Ansatz, but makes in addition the minimum overlap assumption for hadronisation and perturbative systematic uncertainties. The final result, split into the three perturbative schemes, reads as:

$$\begin{array}{ll} \mathit{LogR} & \alpha_s = 0.1205 \pm 0.0020_{exp} \pm 0.0050_{th} \ , \\ \mathcal{O}(\alpha_s^2) & \alpha_s = 0.1157 \pm 0.0018_{exp} \pm 0.0027_{th} \ , \\ \mathit{NLLA} & \alpha_s = 0.1093 \pm 0.0023_{exp} \pm 0.0051_{th} \ . \end{array}$$

OPAL is currently preparing a new analysis on $\alpha_s$ from event shapes and preliminary results were already included in the LEP average. In the last OPAL publication [9] data from LEP up to 189 GeV were combined with lower-energy data from JADE at 35 and 44 GeV. The variables are the differential 2-jet rate (equivalent to $y_3$) and the mean jet multiplicity using the Durham and Cambridge jet-finding algorithms. The combined result is:

$$\alpha_s(M_{\mathrm{Z}}) = 0.1287 \pm 0.0012_{exp} + 0.0034_{th} - 0.0016_{th},$$

where a significantly asymmetric uncertainty is observed for the scale variation.

## 3. power law corrections

Non-perturbative effects in hadronic observables in $e^+e^-$ annihilation are scaling with powers of $1/Q$ and can be described by analytical models of power law corrections [10]. Power corrections in the spirit of these models are related to infrared divergences of the perturbative expansion at low scales. Analytical calculations introduce one additional phenomenological parameter $\alpha_0$,

$$\alpha_0(\mu_{\mathrm{I}}) = \frac{1}{\mu_{\mathrm{I}}} \int_0^{\mu_{\mathrm{I}}} \alpha_s(k) dk \ ,$$

which measures effectively the strength of the coupling up to an infrared matching scale $\mu_{\mathrm{I}}$ of the order of a few GeV. The parameter $\alpha_0$ is expected to be universal and must be determined by experiment, usually in conjunction with $\alpha_s$.

An improved theoretical prediction is obtained by merging perturbative and non-perturbative terms. This yields for event-shape mean values,

$$\langle y \rangle = \langle y_{\mathrm{pert}} \rangle + \langle y_{\mathrm{power}} \rangle,$$

for a generic variable $y$, where the additive power correction term is given by

$$\langle y_{\mathrm{power}} \rangle = c_y \cdot P(\alpha_0)/Q,$$

with a variable-dependent constant $c_y$. In event-shape distributions the power correction appears as a shift of the perturbative spectrum by the same additive term

$$D_y(y) = D_{\mathrm{pert}} \left( y - \frac{c_y \cdot P(\alpha_0)}{Q} \right).$$

For the jet broadenings this shift is not constant but depends on the value of the broadening. In fig. 3 different mean values are shown as function of $\sqrt{s}$ and compared to pure perturbative and power-corrected predictions.

A good description of the data is achieved both for mean values and the central part of distributions. Different groups have analysed often similar datasets including lower energy measurements and determined the two parameters $\alpha_s(M_Z)$ and $\alpha_0(2\ \mathrm{GeV})$ from a simultaneous fit. The combined final results are given in table 1. The value of $\alpha_0(2\ \mathrm{GeV})$ is close to 0.5 and the value

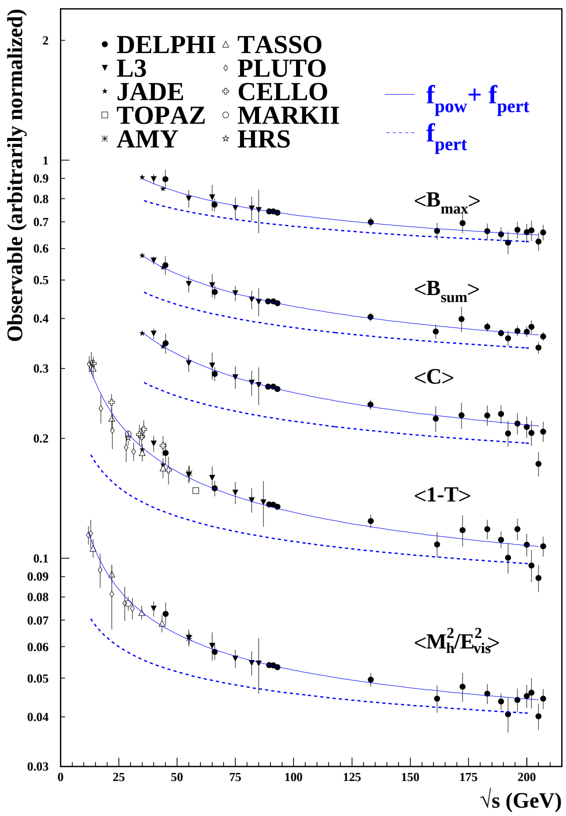


Figure 3. Measurements of event-shape mean values as function of centre-of-mass energy compared to fits including power corrections.

of $\alpha_s(M_Z)$ around 0.114, significantly lower than with standard Monte Carlo corrections for hadronisation. The two parameters in the simultaneous fit are strongly correlated. It is instructive to consider the results in the plane $\alpha_s(M_Z)$ versus $\alpha_0(2\text{ GeV})$, shown for the ALEPH measurements in fig. 4. It appears that the jet broadening variables prefer a significantly higher value of $\alpha_0$ and a lower value of $\alpha_s$, incompatible with thrust and C-parameter, and with $\alpha_s$ determined with Monte Carlo corrections. The large systematic uncertainty for $\alpha_0$ from the jet broadenings is traced back to uncertainties of the perturbatively calculated power correction term, which might

Table 1
Combined results on $\alpha_s(M_Z.)$ and $\alpha_0(2\text{ GeV})$ using different variables. Partly mean values or distributions were analysed.

| | $\alpha_s(M_Z)$ | $\alpha_0(2\text{ GeV})$ |
|---|---|---|
| ALEPH distr. | 0.1112 ± 0.0053 | 0.496 ± 0.101 |
| L3 means | 0.1126 ± 0.0060 | 0.478 ± 0.059 |
| DELPHI distr. | 0.1110 ± 0.0055 | 0.559 ± 0.073 |
| Movilla et al. distr.+means | 0.1171 ± 0.0026 | 0.513 ± 0.050 |

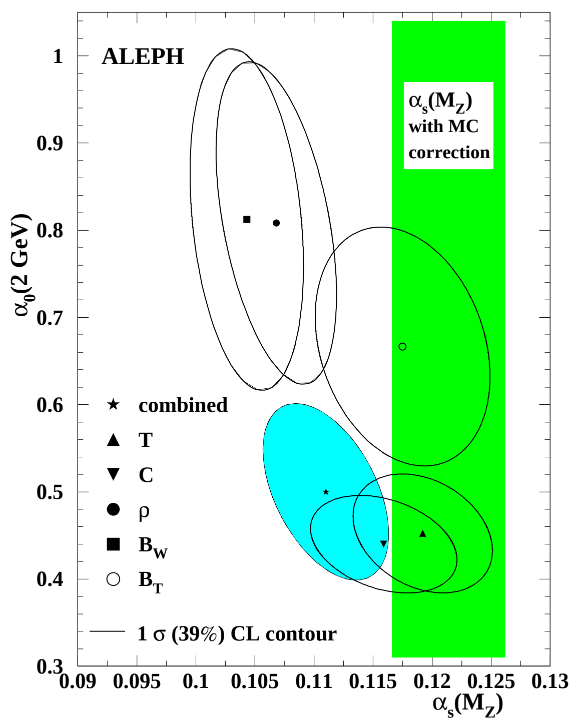


Figure 4. Contours of confidence level for simultaneous measurements of $\alpha_s$ and $\alpha_0$ (ellipses) compared to the combined measurement of $\alpha_s$ using Monte Carlo corrections (shaded band).

be responsible for the large spread between different variables, spoiling the predicted universality of $\alpha_0$. Furthermore the findings of different groups are not always consistent with each other, as demonstrated in fig. 5, where central values and contour ellipses of ALEPH and Movilla et al. are consistent while the value of $\alpha_0$ from DELPHI is much lower and the error ellipse significantly smaller.

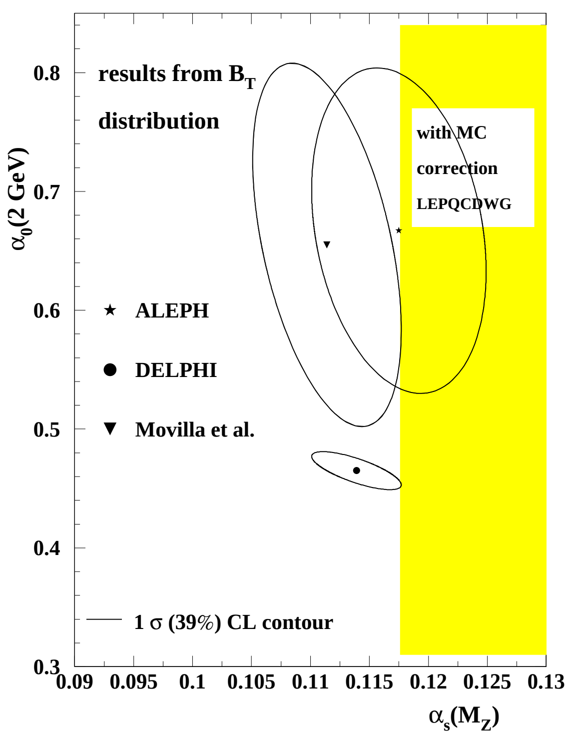


Figure 5. Comparison of confidence level contours from different groups for determinations of $\alpha_s$ and $\alpha_0$ using the total jet broadening.

## 4. 4-jet observables

Traditionally angular 4-jet observables have been used to determine QCD color factors, often in conjunction with a determination of $\alpha_s$ from the 4-jet rate. The precision of the measurements is substantially improved with the advent of NLO $\mathcal{O}(\alpha_s^3)$ calculations. Two analyses have been carried out recently by the ALEPH [11] and OPAL [12] collaborations, which determined simultaneously the colour factor ratios $T_R/C_F$, $C_A/C_F$ and the coupling constant. Using the QCD value of normalisation $T_R$=1/2, ALEPH obtained:

$$
\begin{aligned}
C_A &= 2.93 \pm 0.14_{stat} \pm 0.58_{sys} \, , \\
C_F &= 1.35 \pm 0.07_{stat} \pm 0.26_{sys} \, , \\
\alpha_s(M_Z) &= 0.119 \pm 0.006_{stat} \pm 0.026_{sys} \, .
\end{aligned}
$$

These measurements of the colour factors are excellent agreement with the QCD expectations $C_A = 3$ and $C_F = 4/3$. The OPAL analysis yields

$$
\begin{aligned}
C_A &= 3.02 \pm 0.25_{stat} \pm 0.49_{sys} \, , \\
C_F &= 1.34 \pm 0.13_{stat} \pm 0.22_{sys} \, , \\
\alpha_s(M_Z) &= 0.120 \pm 0.011_{stat} \pm 0.020_{sys} \, ,
\end{aligned}
$$

again in good agreement with the ALEPH measurement. The confidence level contours of the

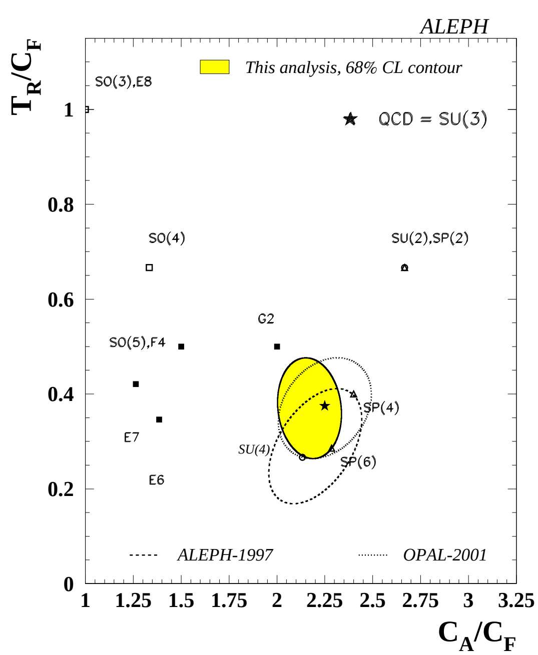


Figure 6. Measurements of the colour factor ratios compared to various gauge group values.

ALEPH and OPAL analyses are shown in fig. 6 in the plane $T_R/C_F$ versus $C_A/C_F$. The measurements confirm the QCD expectation of SU(3).

### 4.1. $\alpha_s$ from the 4-jet rate

Alternatively, setting the colour factors to QCD values, $\alpha_s$ can be determined from certain 4-jet observables like the 4-jet fraction at NLO. A better sensitivity is obtained compared to 3-jet observables, since the leading term is already in $\alpha_s^2$. For the 4-jet rate defined in the Durham and Cambridge schemes also resummed calculations are available, although only for the R matching scheme. ALEPH [11] and DELPHI [13] have presented measurements of $\alpha_s$ using this technique, a new OPAL analysis including also data at LEP2 and from JADE is under way.

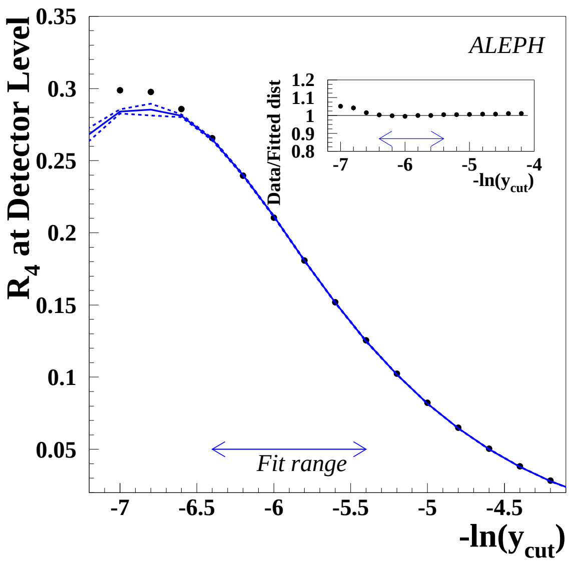


Figure 7. The 4-jet fraction at detector level as function of $\ln y_{\rm cut}$ compared to a fit of the $\mathcal{O}(\alpha_s^3)$+NLLA prediction.

ALEPH measured the 4-jet rate in the Durham at LEP1 only and determined $\alpha_s$ from a fit of the $\mathcal{O}(\alpha_s^3)$+NLLA (R matching scheme) prediction, as shown in fig. 7. The data are well described inside the fit range provided that the renormalisation scale is set the experimentally optimised value $x_\mu^{opt} = 0.73$. The quality of the fit becomes rapidly worse for scales off the optimum. Systematic uncertainties for the 4-jet analysis are not dominated by perturbative but hadronisation uncertainties. This is related to the fact that 4- and 5-jet production are less well described in standard Monte Carlo generators. ALEPH applied a Bayesian method to determine the size of systematic uncertainties, which consists of a de-weighting of models or theories yielding a bad description of the data (large $\chi^2$) by re-scaling the resulting uncertainties. This procedures leads to very small errors.

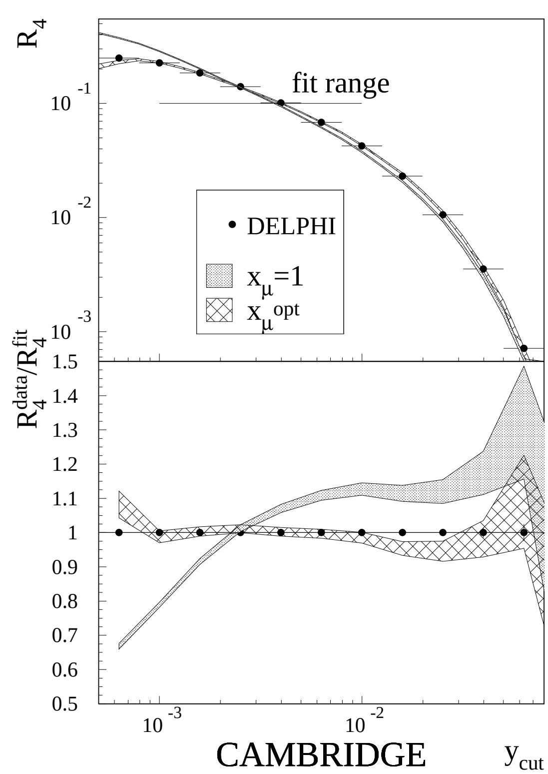


Figure 8. Fits to the 4-jet rate measured at LEP1 for two different scale evaluation methods.

DELPHI investigated the two jet finding schemes Durham and Cambridge and concluded that the Cambridge scheme has smaller systematic uncertainties. Their measurement and fit result are shown in fig. 8, where also the ratio of data over theory is given for $x_\mu = 1$ and $x_\mu^{opt}$. DELPHI used in contrast to ALEPH only the fixed order calculation and found a need for rather small scales, $x_\mu^{opt} = 0.015$ for Durham and $x_\mu^{opt} = 0.015$ for Cambridge. The perturbative systematic uncertainty was estimated by a variation of a factor of 2 around $x_\mu^{opt}$, this gives rise to very small uncertainties. DELPHI confirms that the main systematic uncertainty is associated with the description of $R_4$ in the QCD generators. The 4-jet measurements are summarised in table 2, the ALEPH result is for a direct comparison with DELPHI also given in a non-Bayesian approach for the systematic error. The total uncertainties of about 3 % for these measurements are very competitive compared to methods using 3-jet observables, in particular the perturbative uncertainties are at the 1 % level, similar to full NNLO determinations [15].

Table 2
Measurements of $\alpha_s$ from the 4-jet fraction.

| | DELPHI Cambridge | ALEPH Durham | ALEPH non-Bayesian |
|---|---|---|---|
| $\alpha_s(M_Z)$ | 0.1175 | 0.1170 | 0.1170 |
| exp. | 0.0009 | 0.0008 | 0.0008 |
| had. | 0.0027 | 0.0004 | 0.0021 |
| pert. | 0.0007 | 0.0009 | 0.0016 |
| tot. | 0.0030 | 0.0013 | 0.0027 |

## 5. Conclusion

A wealth of measurements of hadronic observables has been provided in 11 years of data taking at the LEP collider. These measurements allowed the LEP collaborations to perform detailed tests of perturbative QCD and determinations of the fundamental parameters. A collection of the most important results is shown in fig. 9. All collaborations have provided measurements of $\alpha_s$ from event-shape distributions and combined individual results from various variables at energies from $M_Z$ to 209 GeV. The systematic uncertainties are dominated by missing higher orders and in order to match the experimental accuracy of 1 % NNLO calculations are required. Event-shapes have further been investigated in the context of power-law corrections. Simultaneous fits have been carried out to determine the non-perturbative parameter $\alpha_0$ and $\alpha_s$. While the qualitative description of the data is good, the quantitative interpretation is not unambiguous since the value of $\alpha_s$ with power corrections is significantly lower than with Monte Carlo corrections for hadronisation. Furthermore, the predicted universality of $\alpha_0$ is verified at the level of 20% of its precision only. In particular the jet broadening variables need to be further investigated.

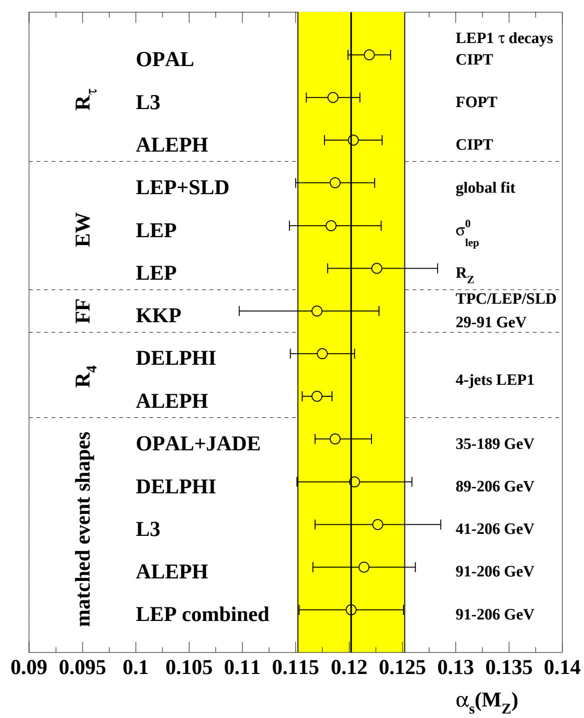


Figure 9. Summary of measurements of $\alpha_s(M_Z)$ at LEP using different methods and observables.

Complete NLO calculations for 4-jet production improved significantly the determinations of the QCD colour factors, which are found to be in good agreement with SU(3). New measurements of $\alpha_s$ from the 4-jet rate using these calculations have been presented. The precision of this method is very good and yields a 1 % uncertainty for missing higher orders. The total uncertainty is of 3 %, dominated by model uncertainties. The 4-jet method is comparable in precision with determinations using the fully inclusive observables $R_Z$ and $R_\tau$, for which complete NNLO calculations are available.

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